Course Number and Section

Rough Path Theory (Fall 2020)

Instructor Information

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Dear students, my name is Vlad Margarint and I will be the instructor for the Rough Path Theory class. This subject is very rich in both pure Mathematical content as well as applications!

Course Information

- Fall 2020 14 weeks 01-09-2020- 20-12-2020.
- 3 credits

The content of the class is divided into several subsections including:Riemann-Stieltjes Theory of Integration, Young theory of integration with examples, Picard's Fix Point Theorem, Signature of a path of finite p-variation for $p<2$, Tensor algebra and its propreties, Group-like elements and Shuffle Products, Definition of Rough Paths, Extension Theorem, Geometric Rough Paths and applications, Rough Differential Equations, Terry Lyons' Universal Limit Theorem. As a historical account, the theory was introduced by Terry Lyons in 1998 in 'Revista Mathematica Iberoamericana'. The theory provides a deterministic alternative to Ito's theory of stochastic integral, and provides new insights into differential equations driven by canonical stochastic processes such as Brownian motion. Also, the theory goes beyond the integration of semimartingales and provides analytical tools to study the mathematical properties of the solutions of such equations. The field has an enormous range of applications from Stochastic Analysis, Machine Learning and Data Analysis, Dynamical systems etc. In 2014, Martin Hairer received the Fields Medal-highest honor in Mathematics for extending Rough Path Theory to the study of Stochastic Partial Differential Equations (SPDEs) by introducing the Theory of Regularity Structures.
I hope you will enjoy the ride!

- **Prerequisites**: Admissible students should have passed the courses 'Honors Analysis 1 and 2', Probability and Martingales' and ' Stochastic Calculus' offered at NYU Shanghai/ NYU Courant, or some equivalent course elsewhere.

- Weekly Contact Hours:
    - Lectures: Twice weekly Monday 10:00-11:00, Wednesday 10:00-11:00
      Office Hours: At 18:00 every Wednesday in Room 1133-S. In certain circumstances office hours may be scheduled at other times by appointment, feel free to reach out by email.

Course Learning Outcomes

Upon completion of this course, students will be able to:

- Apply techniques of Riemann-Stieltjes and Young theory of integration to a variety of contexts with applications in Stochastic Calculus.
- Work with the Tensor Algebra (and with tensor products) and with shuffle products.
- Analyze iterated integrals and collections of such objects (the signature) and apply them in various contexts including Data Science.
- Work with Rough Paths and apply the techniques to a variety of problems from Stochastic Analysis in order to provide new insights (using analytic techniques different from the typical Stochastic Calculus class).
- Analyze Rough Differential Equations and obtain analytic information about their solutions.